Simple eigenvalues for the Laplace operator in a domain with a small hole.

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Let $\mathbb{I}^o$ be a bounded open domain of $\mathbb{R}^n$. Let $\nu_{\mathbb{I}^o}$ denote the outward unit normal to $\partial\mathbb{I}^o$. We assume that the Steklov problem $\Delta u = 0$ in $\mathbb{I}^o$, $\frac{\partial u}{\partial \nu_{\mathbb{I}^o}} = \lambda u$ on $\partial\mathbb{I}^o$ has a simple eigenvalue $\tilde{\lambda}$. Then we consider an annular domain $\mathbb{A}(\epsilon)$ obtained by removing from $\mathbb{I}^o$ a small cavity of size $\epsilon > 0$, and we show that under proper assumptions there exists a real valued and real analytic function $\hat{\lambda}(\cdot,\cdot)$ defined in an open neighborhood of $(0,0)$ in $\mathbb{R}^2$ and such that $\hat{\lambda}(\epsilon, \delta_{2,n}\epsilon\log\epsilon)$ is an eigenvalue for the Steklov problem $\Delta u = 0$ in $\mathbb{A}(\epsilon)$, $\frac{\partial u}{\partial \nu_{\mathbb{A}(\epsilon)}} = \lambda u$ on $\partial\mathbb{A}(\epsilon)$ for all $\epsilon > 0$ small enough, and such that $\hat{\lambda}(0,0) = \tilde{\lambda}$. Here $\nu_{\mathbb{A}(\epsilon)}$ denotes the outward unit normal to $\partial\mathbb{A}(\epsilon)$, and $\delta_{2,2} \equiv 1$ and $\delta_{2,n} \equiv 0$ if $n \geq 3$. Then related statements have been proved for corresponding eigenfunctions.

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